

次の式を簡単にせよ。

$$(1) \sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\dots}}}}$$

$$(2) \sqrt[16]{2207-\frac{1}{2207-\frac{1}{2207-\dots}}}$$

(解)

$$(1) \sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\dots}}}} = x \text{ とおくと, } x^2 = 6 + \sqrt{6+\sqrt{6+\sqrt{6+\dots}}} = 6 + x$$

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

$x > 0$  より

$$\therefore x = 3 \dots (\text{答})$$

$$(2) \sqrt[16]{2207-\frac{1}{2207-\frac{1}{2207-\dots}}} = x \text{ とおくと, } x^{16} = 2207 - \frac{1}{2207-\frac{1}{2207-\dots}} > 1 \text{ である。}$$

$$2207 - x^{16} = \frac{1}{2207 - \frac{1}{2207 - \dots}} = \frac{1}{2207 - (2207 - x^{16})} = \frac{1}{x^{16}} \text{ より}$$

$$x^{32} - 2207x^{16} + 1 = 0$$

$$x^{16} = \frac{2207 \pm \sqrt{2207^2 - 4}}{2}$$

$$x^{16} > 1 \text{ より}$$

$$\begin{aligned} x^{16} &= \frac{2207 + \sqrt{2209 \cdot 2205}}{2} = \frac{4414 + 2\sqrt{2209 \cdot 2205}}{4} = \left( \frac{\sqrt{2209} + \sqrt{2205}}{2} \right)^2 = \left( \frac{47 + \sqrt{2205}}{2} \right)^2 = \left( \frac{94 + 2\sqrt{49 \cdot 45}}{4} \right)^2 \\ &= \left( \frac{\sqrt{49} + \sqrt{45}}{2} \right)^4 = \left( \frac{7 + \sqrt{45}}{2} \right)^4 = \left( \frac{14 + 2\sqrt{9 \cdot 5}}{4} \right)^4 = \left( \frac{\sqrt{9} + \sqrt{5}}{2} \right)^8 = \left( \frac{3 + \sqrt{5}}{2} \right)^8 = \left( \frac{6 + 2\sqrt{5}}{4} \right)^8 = \left( \frac{1 + \sqrt{5}}{2} \right)^{16} \end{aligned}$$

$$\therefore x = \frac{1 + \sqrt{5}}{2} \dots (\text{答})$$

(2015/8/4 時岡)