

次を証明せよ。

$$(1) \frac{1}{1 \cdot 2} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{(2n-1)(2n+1)} = \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n}$$

$$(2) 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots = \log 2$$

(証明)

$$\begin{aligned} (1) \text{ 左辺} &= \left(1 - \frac{1}{2}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \cdots + \left(\frac{1}{2n-1} - \frac{1}{2n}\right) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{2n} - 2\left(\frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2n}\right) \\ &= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{2n} - \left(1 + \frac{1}{2} + \cdots + \frac{1}{n}\right) = \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n} = \text{右辺} \quad \blacksquare \end{aligned}$$

$$(2) 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots = \lim_{n \rightarrow \infty} \left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots + \frac{1}{2n-1} - \frac{1}{2n}\right) = \lim_{n \rightarrow \infty} \left(\frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n}\right) = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \frac{1}{1 + \frac{k}{n}}$$

$$= \int_0^1 \frac{dx}{1+x} = [\log(1+x)]_0^1 = \log 2 \quad \blacksquare$$

(2019/2/24 時岡)